International Journal of
Materials
& Structural Reliability

International Journal of Materials & Structural Reliability Vol.3, No.2, September 2005, 105 - 116

Time Dependent Reliability Analysis of the Serviceability of Corrosion Affected Concrete Structures

Chun Qing Li

Division of Civil Engineering, University of Dundee, DD1 4HN, UK

Abstract

This paper attempts to present a reliability-based methodology for serviceability assessment of corrosion affected concrete structures using the performance criterion of structural deflection. A stochastic model for deflection is developed based on the data produced from an experiment designed to investigate the effect of steel reinforcement corrosion on structural deterioration. A time-dependent reliability method is employed to quantify the probability of serviceability failure so that the time for the structure to be unserviceable due to excessive deflection can be determined with confidence. Factors that affect the serviceability of corrosion affected concrete structures are also studied using two sensitivity analysis techniques. A merit of the proposed methodology is that the structural assessment is directly related to design criterion used by structural engineers. It is found that the deterioration function propsoed in the paper can best represent the effect of corrosion on deflection deterioration. A reliability-based assessment for structural serviceability can assist structural engineers and asset managers to develop a risk-informed and cost-effective strategy in the management of corrosion-affected concrete infrastructure.

Keywords: Reinforcement Corrosion; Concrete Structures; Deflection; Serviceability failure; Sensitivity.

---

1. Introduction

Reinforcement corrosion induced structural failure does not necessarily imply structural collapse but in most cases manifest the loss of structural serviceability, as characterized by concrete spalling and the excessive deflection of concrete members (Broomfield 1997, Chaker 1992, Schiessl 1988). Practical experience (Dhir and McCarthy 1999) and experimental observations (Li 2003) suggest that corrosion affected concrete structures deteriorate at different rates as measured by strength and serviceability with the latter deteriorating faster. As an example of concrete flexural members shown in Figure 1 (Li 2003), at the time that the structural strength deteriorates to about 90% of its initial strength, its serviceability (represented by deflection) deteriorates to about 60% of its original state under the same test conditions. The reason for this is attributed to the nature of the problem; the corrosion products exert an expansive pressure on concrete. Due to the low tensile strength of concrete, this expansive pressure leads to a domino effect: concrete cracking, spalling and de-bonding between the reinforcement and concrete. All these effects are prominent once corrosion actively propagates in concrete structures. As a consequence, the stiffness of the structure reduces and the deflection increases.

---

[*] Corresponding author.
E-mail : c.q.li@dundee.ac.uk

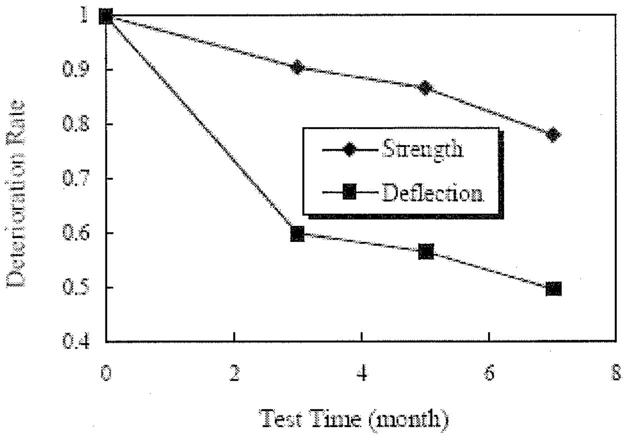


Fig. 1 Structural deterioration under different limit states

The more severe deterioration of structural serviceability explains why so many concrete structures are seen as "badly" deteriorated (e.g., mass concrete spalling) but still structurally sound. Since the safety factors (load and resistance factors) used in design for structural strength are usually larger than those for serviceability (due to the paramount importance of structural safety) the actual risk in loss of strength is smaller than that in serviceability. This gives rise to the need for a reliability analysis on corrosion induced structural deflection. On the other hand, since the costs of repairs are usually high for concrete structures (in addition to the inconvenience to the public due to interruptions during repairing), it is of practical importance to accurately predict the time for repairs for deteriorated structures so as to achieve cost-effectiveness in the asset management of concrete structures. It is in this regard that a time-dependent reliability analysis is in order.

The intention of this paper is to present a reliability-based methodology for serviceability assessment of corrosion affected concrete structures, embodied with the deflection of flexural members. A stochastic model for structural deflection is developed based on the experimental data produced from thirty concrete flexural members with structurally significant size and subjected to simultaneous corrosion intrusion and service loads. A time-dependent reliability method is employed to quantify the probability of serviceability failure so that the time for the structure to be unserviceable, due to excessive deflection, can be determined with confidence. Factors that affect the serviceability of corrosion affected concrete structures are also studied using two sensitivity analysis techniques. A merit of the proposed methodology is that the structural assessment is directly related to design criterion used by structural engineers. The methodology presented in the paper can serve as a tool for structural engineers and asset managers to make decisions with regard to the serviceability of corrosion affected concrete structures. Timely maintenance and repairs have the potential to prolong their service life.

2. Formulation of Structural Serviceability

According to design codes and standards, e.g., ACI 318 (1999) and BS 8110 (1997), one of the performance criteria related to the serviceability of practical concrete structures is to control the deflection under an acceptable limit. In the theory of structural reliability, this criterion can be expressed in the form of a limit state function as follows

$$G(L,\Delta,t) = L(t) - \Delta(t) \qquad (1)$$

where denotes structural deflection (load effect) at time t and $L(t)$ is a critical limit for structural deflection (resistance) increases with time even under the constant load due to corrosion induced concrete cracking and spalling. $L(t)$ may change with time although in most practical applications it can a constant prescribed in design codes and standards. With the limit state function of Equation (1), the probability of serviceability failure due to excessive deflection, p_d, can be determined by

$$p_d(t) = P[G() \le 0] = P[\Delta(t) \ge L(t)] \tag{2}$$

As can be seen, Equation (2) represents a typical upcrossing problem, which can be dealt with using time-dependent reliability methods (Melchers 1999). Time-dependent reliability problems are those in which either all or some of basic random variables are modeled as stochastic processes. For structural serviceability problems involving stochastic process of structural deflection, $\Delta(t)$, the structural serviceability depends on the time that is expected to elapse before the first occurrence of the stochastic process, $\Delta(t)$, upcrossing a critical limit (the threshold), $L(t)$, sometime during the service life, $[0, t_L]$, of the structure. Equivalently, the probability of the first occurrence of such an excursion is the probability of serviceability failure, $p_d(t)$, during that time period. This is known as "first passage probability" and can be determined by (Melchers 1999)

$$p_d(t) = 1 - [1 - p_d(0)]e^{-\int_0^t \upsilon t} \tag{3}$$

where is the probability of serviceability failure due to excessive deflection at time $t = 0$ and is the mean rate for the deflection process, $\Delta(t)$, to upcross the threshold, $L(t)$. In many practical problems, the mean upcrossing rate, υ, is very close to zero so that Equation (3) can be approximated as follows

$$p_d(t) = p_d(0) + \int_0^t \upsilon d\tau \tag{4}$$

The upcrossing rate in Equation (4) can be determined from the Rice formula (e.g., Melchers 1999)

$$\upsilon = \upsilon_L^+ = \int_{\dot{L}}^{\infty} (\dot{\Delta} - \dot{L}) f_{\Delta\dot{\Delta}}(L, \dot{\Delta}) d\dot{\Delta} \tag{5}$$

where υ_L^+ is the upcrossing rate of the deflection process $\Delta(t)$ relative to the threshold L, $\dot{L}$ is the slope of L with respect to time t, $\dot{\Delta}(t)$ is the time derivative process of $\Delta(t)$ and $f_{\Delta\dot{\Delta}}()$ is the joint probability density function for Δ and $\dot{\Delta}$. An analytical solution to Equation (5) has been derived for a deterministic threshold L in Li and Melchers (1993) and is expressed as

$$\upsilon_L^+ = \frac{\sigma_{\dot{\Delta}|\Delta}}{\sigma_\Delta} \phi\left(\frac{L - \mu_\Delta}{\sigma_\Delta}\right) \left\{ \phi\left(\frac{\dot{L} - \mu_{\dot{\Delta}|\Delta}}{\sigma_{\dot{\Delta}|\Delta}}\right) - \frac{\dot{L} - \mu_{\dot{\Delta}|\Delta}}{\sigma_{\dot{\Delta}|\Delta}} \Phi\left(-\frac{\dot{L} - \mu_{\dot{\Delta}|\Delta}}{\sigma_{\dot{\Delta}|\Delta}}\right) \right\} \tag{6}$$

where φ( ) and Φ( ) are standard normal density and distribution functions respectively, μ and σ denote the mean and standard deviation of Δ and $\dot{\Delta}$, represented by subscripts and " | " denotes the condition. For a given Gaussian stochastic process with mean function, $\mu_\Delta(t)$, and auto-covariance function, $C_{\Delta\Delta}(t_i, t_j)$, the variables in Equation (6) can be determined, according to the theory of stochastic processes (see, e.g., Papoulis (1965) and Melchers (1999)), as follows

$$\mu_{\dot{\Delta}|\Delta} = E[\dot{\Delta} \mid \Delta = L] = \mu_{\dot{\Delta}} + \rho \frac{\sigma_{\dot{\Delta}}}{\sigma_\Delta}(L - \mu_\Delta) \tag{7a}$$

$$\sigma_{\dot{\Delta}|\Delta} = [\sigma_{\dot{\Delta}}^2(1 - \rho^2)]^{1/2} \tag{7b}$$

where

$$\mu_{\dot{\Delta}} = \frac{d\mu_{\Delta}(t)}{dt} \tag{7c}$$

$$\sigma_{\dot{\Delta}} = \left[\frac{\partial^2 C_{\Delta\Delta}(t_i,t_j)}{\partial t_i \partial t_j} \Big|_{i=j} \right]^{1/2} \tag{7d}$$

$$\rho = \frac{C_{\Delta\dot{\Delta}}(t_i,t_j)}{[C_{\Delta\Delta}(t_i,t_i) \cdot C_{\dot{\Delta}\dot{\Delta}}(t_j,t_j)]^{1/2}} \tag{7e}$$

and the cross-covariance function is

$$C_{\Delta\dot{\Delta}}(t_i,t_j) = \frac{\partial C_{\Delta\Delta}(t_i,t_j)}{\partial t_j} \tag{7f}$$

Since it is unlikely that the deflection of a RC structure exceeds a critical limit at the beginning of its service, the probability of serviceability failure due to excessive deflection at $t = 0$ is zero, i.e., $p_d(0) = 0$. Also, since in most practical applications, the critical limit for deflection $L(t)$ is a constant, δ, as prescribed in design codes and standards, the solution to Equation (4) can be expressed, after substituting Equation (6) into Equation (4) and replacing $L(t)$ with δ, as

$$p_d(t) = \int_0^t \frac{\sigma_{\dot{\Delta}|\Delta}(t)}{\sigma_{\Delta}(t)} \phi\left(\frac{\delta - \mu_{\Delta}(t)}{\sigma_{\Delta}(t)} \right) \left\{ \phi\left(-\frac{\mu_{\dot{\Delta}|\Delta}(t)}{\sigma_{\dot{\Delta}|\Delta}(t)} \right) + \frac{\mu_{\dot{\Delta}|\Delta}(t)}{\sigma_{\dot{\Delta}|\Delta}(t)} \Phi\left(\frac{\mu_{\dot{\Delta}|\Delta}(t)}{\sigma_{\dot{\Delta}|\Delta}(t)} \right) \right\} d\tau \tag{8}$$

At a time that $p_d(t)$ is greater than a maximum acceptable risk in terms of the probability of serviceability failure, p_a , it is the time the structure becomes unserviceable. This can be determined from the following

$$p_d(T_d) \geq p_a \tag{9}$$

where T_d denotes the time the structure becomes unserviceable due to excessive deflection. In principle, p_a can be determined from risk-cost optimization of the structure during its whole service life. This is beyond the scope of the paper and will not be discussed herein but can be referred to, e.g., Li (1995) and Thoft-Christensen and Sorensen (1987).

3. Model of Corrosion Induced Deflection

For Equation (8) to be of practical use, i.e., determining the probability of serviceability failure due to excessive deflection over time, the key is to develop a stochastic model for deflection. For corrosion affected concrete structures, the deflection increases even under the constant load due to corrosion induced concrete cracking and spalling, which reduce the stiffness of the structural member, and importantly, the de-bonding between the reinforcement and concrete. To account for this effect, a deterioration function is introduced so that structural deflection can be modeled as

$$\Delta(t) = \varphi(t)\Delta_0 \tag{10}$$

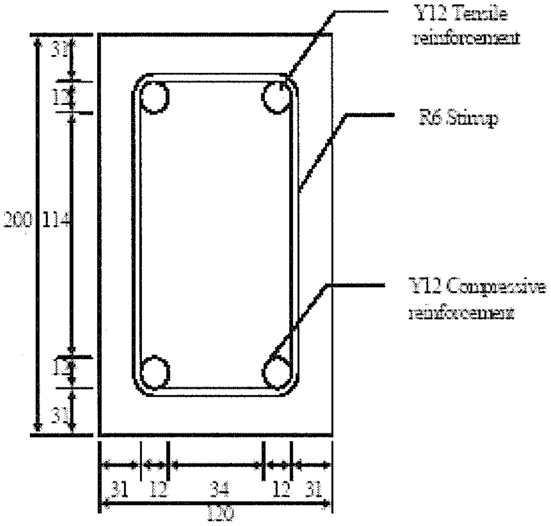
where $\varphi(t)$ is the deterioration function for deflection and Δ_0 is the initial deflection of the structure. In Equation (10), $\varphi(t)$ is not only random but also time-variant (increasing function with time) due to the nature of structural deterioration. One advantage of the deflection model of Equation (10) is the use of relative (i.e., $\varphi(t) = \Delta(t) / \Delta_0$) rather than absolute values for quantifying deflection deterioration. This relative measure facilitates the normalization of data collected from different type of structures (i.e., simple support and cantilever), thereby maximizing the use of data, which is in general scarce. This is particularly advantageous for statistical analysis of the data due to the increased population.

3.1 Data production

As may be appreciated, the deterioration process induced by reinforcement corrosion in concrete structures is intricate and complex. In view of the current state of knowledge and understanding of corrosion propagation and its effect on structural deterioration, both strength and deflection (e.g., ACI C365 2002, Andrade, et al. 1996, Bentz, et al. 1999, Frangopol, et al. 1997, Liu and Weyers 1998, Melchers 2001, Otsuki, et al. 2000), it seems that effort to deriving a theoretical model for deflection deterioration, i.e., the deterioration function $\varphi(t)$ in Equation (10), may not lead to a satisfactory solution. In this paper, the effort is devoted to developing an empirical model for the deterioration function. This approach can be justified when the historical development of the theories of reinforced concrete structures is examined (e.g., Mirza, et al. 1979), in which almost all design formulae (e.g., deflection and sectional stiffness) are based on large quantity of experiments. In this paper, test data produced from thirty concrete flexural members with structurally significant size and subjected to simultaneous corrosion intrusion and service loads are used to derive the deterioration function, $\varphi(t)$. Detailed information of the test regime and testing methodology has been published (Li 2001, 2003) and hence will not be repeated herein except for a brief description as follows.

To investigate the effect of reinforcement corrosion on structural capacity deterioration (including both the strength and deflection), a comprehensive experiment on large scale concrete cantilever beans was undertaken by Li (2001, 2003). An overview of test set up and a typical cross section of the test specimens is shown in Figure 2. For the purposes of developing stochastic models, two replicas were used for each test specimen to generate a minimum of four sets of data at cantilevers (see Figure 2), necessary to form a basis for a statistical study. The total number of test specimens was thirty, consisting of different concrete compositions, e.g., water cement ratio and cement type. Test specimens were under simultaneous saltwater spray and service loads simulated in a controlled environmental chamber built specifically for the designed experiment (Li 2001). To achieve measurable deterioration within a reasonable work period (Francois and Castel 2001), the experiment was conducted under accelerated conditions for the corrosion process. This requires the calibration tests to address and verify the applicability of the data and subsequent models acquired under the accelerated conditions to conditions typical of service. Based on the calibration tests, an acceleration factor was derived to transform the accelerated (test) time to actual time (Li 2000, 2003). Once applied, the loads were kept constant on the specimens. The deflection was measured at the cantilever ends of the specimen so that the combined effect of corrosion on the deflection deterioration due to concrete cracking, spalling and de-bonding was included, but the effect of creep and shrinkage was excluded in processing data through calculation (Gilbert 1988). Typical test results, as expressed in mean and coefficient of variation (COV) of the deterioration function (i.e., $\varphi(t) = \Delta(t) / \Delta_0$, where Δ_0 = 10.68 mm for the test specimens shown in Figure 2) are shown in Figures 3 and 4.

(a) Test set-up for corrosion induced structural deterioration (photo)



(b) Detailed cross-section of test specimens
(Specimens are made of normal concrete with water cement ratio in the range of 0.45-0.6. "Y" stands for rebars with $f_y = 400$ MPa and "R" with $f_y = 250$ MPa)

Fig. 2 Test used in data production

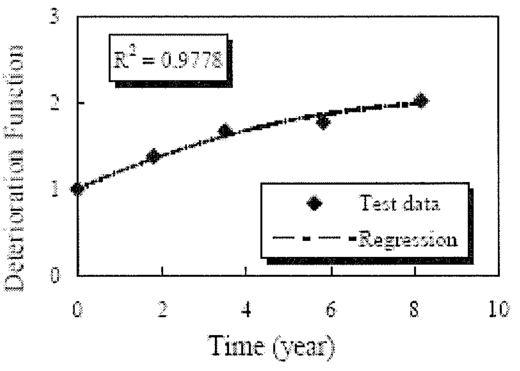


Fig. 3 Mean function of deflection deterioration φ
(R = coefficient of correlation)

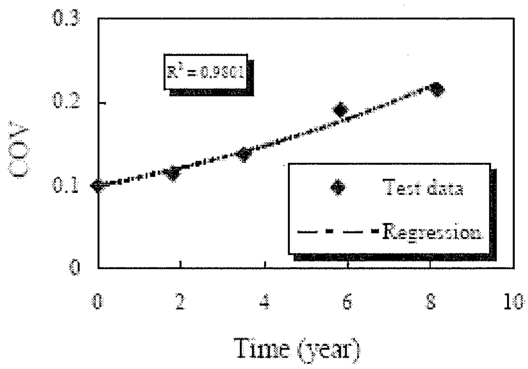


Fig. 4 COV function of deflection deterioration φ
(R = coefficient of correlation)

3.2 Stochastic model

With these test data, a model of deterioration function for deflection can be derived. Due to the randomness of deterioration function and its time-variant nature it is justifiable to model it as a stochastic process, quantified by a mean function, $\mu_\varphi(t)$, and an auto-covariance function $C_{\varphi\varphi}(t_i,t_j)$. From regression analysis of the test data in Figure 3, the mean function of φ can be expressed as

$$\mu_\varphi(t) = -0.01t^2 + 0.22t + 1 \tag{11a}$$

The coefficient of correlation for this regression is 0.99. Likewise, the COV function $V_\varphi(t)$ of φ can be expressed, based on regression analysis of the test data in Figure 4, as

$$V_\varphi(t) = 0.1e^{0.1t} \tag{11b}$$

Again the coefficient of correlation for this regression is satisfactory. Thus the auto-covariance function for, $\varphi(t)$, $C_{\varphi\varphi}(t_i,t_j)$, can be expressed as

$$C_{\varphi\varphi}(t_i,t_j) = \rho_\varphi V_\varphi(t_i)V_\varphi(t_j)\mu_\varphi(t_i)\mu_\varphi(t_j) \tag{11c}$$

where ρ_φ is (auto-) correlation coefficient for $\varphi(t)$ between two points in time t_i and t_j. With $\mu_\varphi(t)$ and $C_{\varphi\varphi}(t_i,t_j)$, Equations (7a) to (7f) can be used to determine other statistical parameters of $\varphi(t)$.

It is acknowledged that thirty test specimens are not sufficient to produce convincing data for the development of deterioration model (i.e., Equation (11)) since there are many factors affecting the deflection deterioration. In the wake of lack of existing models for deflection deterioration (see above references), Equation (11) may serve as a starting point in developing models of deflection deterioration. Certainly it can be used as an example to demonstrate the reliability analysis of the corrosion effect on structural deflection. It needs to be noted that, in practical application, the deflection deterioration model of Equation (11) should be derived based on data obtained from site-specific measurement on the structure to be assessed.

With the model of deflection deterioration and a code-prescribed deflection limit $\delta = 8.33$ (span/150) mm, the probability of serviceability failure due to excessive deflection can be computed using Equation (8) and the results are shown in Figure 5. As can be seen from Figure 5, the correlation between the deflection at two points in time (i.e., ρ_φ) affects the deflection failure considerably. This makes sense since the corrosion induced deflection is attributed to many factors, such as corrosion rate, concrete cracking, de-bonding and the geometry not only at a cross-section but also along the span of the structure. These factors are inter-related at different points in time so that the resultant effect, i.e., the deflection is correlated at different times. Figure 6 shows that different acceptable limits for the deflection would lead to different probability of serviceability failure. This is self-evident.

Finally, as a complete picture of serviceability assessment of corrosion affected concrete structures, the time for the structure to be unserviceable, i.e., T_d, due to corrosion induced excessive deflection, can be determined for a given acceptable risk. This is straightforward. For example, from Equation (9) it can be obtained that T_d = 8.95 years for p_a = 0.1 and δ = span/150. If there is no intervention during the service period of (0, 8.95) for the structure of concern, such as maintenance and repairs, T_d represents the time for interventions or the end of service for the structure, based on the performance criterion of deflection. The information of T_d (i.e., time for interventions) is of significant practical importance to structural engineers and asset mangers of concrete infrastructure in decision-making with regard to its repairs, strengthening and/or rehabilitation which are usually dependent on the budget situation of the day. Therefore, when to intervene is the first question to decision-makers.

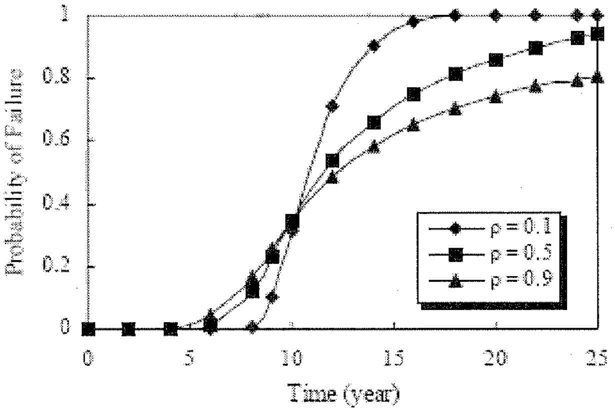


Fig. 5 Probability of deflection failure for different correlation coefficient ρ_φ

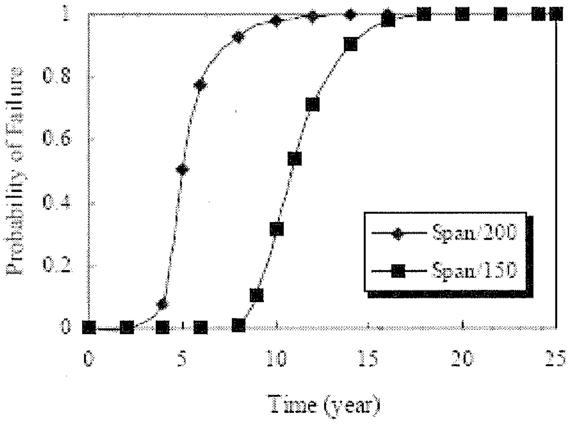


Fig. 6 Probability of deflection failure for different critical limit δ (ρ_φ = 0.1)

4. Sensitivity Analysis

As may be appreciated, statistical information is essential to reliability analysis. In view of lack of full statistical information on basic random variables, it is of interest to identify those random variables whose randomness and distribution parameters affect the deflection most so that the data production, such as tests, can focus on those variables. This can be achieved through sensitivity analysis with respect to design variables of the structure using a sensitivity factor (Novak, et al. 1993). The sensitivity factor is a measure of the importance of input variables and/or parameters to the output. Practical methods of sensitivity analysis for reliability problems are based on first order reliability (FOR) theory, such as Madsen, et al (1986) and Ditlevsen and Madsen (1996), in which the probability of serviceability failure, p_d, is expressed in terms of a reliability index, β, using the well known relationship (Melchers 1999)

$$p_d(t) = \Phi[-\beta(t)] \tag{12}$$

In computing the reliability index, the basic random variables **X** are transformed into standardized normal space **U** and the limit state function, G(**X**, t) (also known as safety margin), transformed to g(**U**, t) (Melchers 1999).

As proposed in Ditlevsen and Madsen (1996), the effect of the randomness of design variables on the probability of serviceability failure can be measured by an omission sensitivity factor. According to Ditlevsen and Madsen (1996), the omission sensitivity factor with respect to a random variable u_i can be determined by

$$\zeta_{u_i}(t) = \frac{\beta(t)\,|_{U_i(t)=u_i}}{\beta(t)} = \frac{1-\alpha_i u_i / \beta(t)}{\sqrt{1-\alpha_i^2}} \tag{13}$$

where α is the normal unit vector to the limit state surface g(**U**, t) at the checking point $\mathbf{u}^*$ and time t (Melchers 1999). As can be seen the omission sensitivity factor expresses relative error in the value of reliability index β if an input random variable is replaced by a fixed value (i.e., treated as a deterministic variable). Thus for those random variables with small relative errors, i.e., $\zeta_{u_i} \approx 1$, it may be appropriate to treat them as deterministic variables if the full statistical information of them is not available.

For the serviceability assessment of RC flexural members, the deflection process, $\Delta(t)$, can be expressed in terms of design variables as

$$\Delta(t) = f(B, E_c, H, \varphi) \tag{14}$$

where B and H are width and depth of the cross-section of the structural member and E_c is the elastic modulus of concrete.

Using statistical values of basic design variables given in Table 1, the omission sensitivity factor can be computed and shown in Figure 7. It can be seen from the figure that the deterioration function φ (Phi in the figure) is the most important variable that affects the deflection of corrosion induced deteriorated structures. This is not only consistent with practical experience but also understandable since the corrosion, as represented by φ, is the source of the deterioration, causing concrete cracking, spalling and importantly de-bonding between the reinforcement and concrete. All these factors reduce the deflection. Therefore full statistical information of φ is essential to structural serviceability assessment. This may be the justification for more research, in particular, experimental research on the effect of corrosion on structural deterioration. From Figure 7 it can also be seen that the height of the cross-section of flexural members, *H*, is important to structural serviceability (deflection) at the beginning. But this importance decreases over time once corrosion actively propagates. Figure 7 clearly shows that some design variables may be treated as deterministic variables, such as the width of the cross-section of flexural members (i.e., *B*) and the material property E_c.

The effect of input parameters on structural serviceability can be measured by a parameter sensitivity factor (Bjerager and Krenk, 1989, Madsen, et al 1986). These parameters, denoted collectively as **θ**, enter the problem either in establishing limit state functions or in transforming design variables to standardized normal variables through $\mathbf{u} = T(\mathbf{x};\theta)$. The parameter sensitivity factor with respect to parameter θ_i can be determined by (Madsen, et al 1986)

$$\gamma_{\theta_i} = \frac{\partial \beta}{\partial \theta_i} = \frac{1}{\beta} \mathbf{u}^{*\mathrm{T}} \frac{\partial T(\mathbf{x}^*;\theta_0)}{\partial \theta_i} \tag{15}$$

where the reliability index is computed at design point $\mathbf{u}^*$ and the value θ_0 used in transformation. As may be appreciated the mean and standard deviation (i.e., μ and σ) are the most important parameters of random variables in the context of FOR theory. Thus the sensitivity of structural serviceability to these two parameters is evaluated and the results are shown in Figures 8 and 9. Again, as can be seen, both the mean and standard deviation of the deterioration function φ (Phi in the figures) are important to structural serviceability. The negative sign indicates that the effects are inversely sensitive, meaning that the increase in both μ and σ results in the decrease of structural serviceability (i.e., β). Again this makes sense both theoretically in terms of limit state function of Equation (2) and practical experience. As can also be seen, the effect of the mean of deterioration function φ on structural serviceability decreases with time while that of the standard deviation of φ changes little with time.

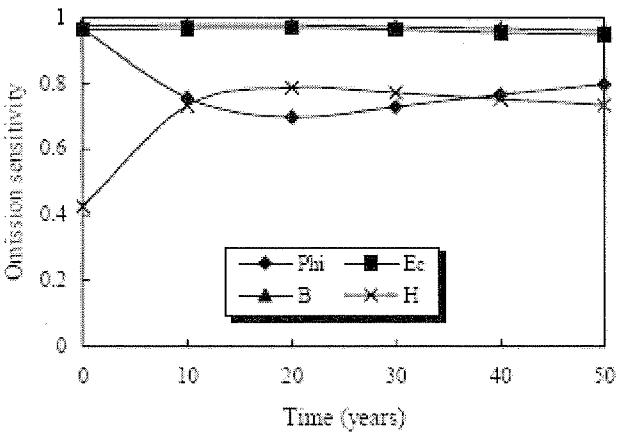


Fig. 7 Sensitivity to the randomness of design variables

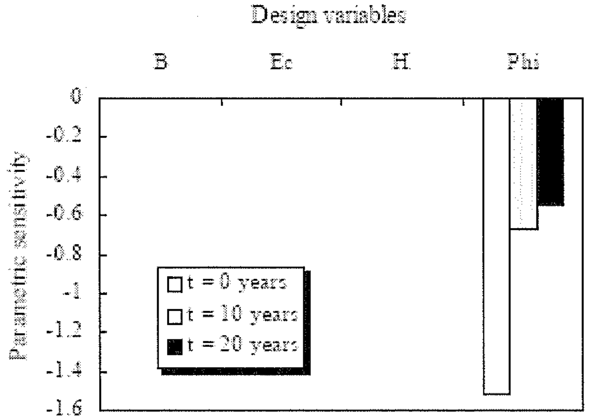


Fig. 8 Sensitivity to the mean of design variables

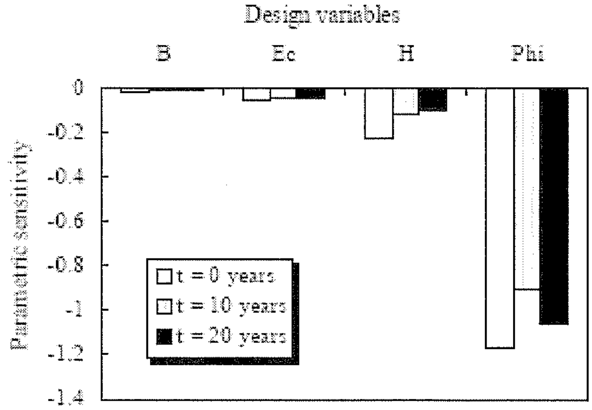


Fig. 9 Sensitivity to the standard deviation of design variables

Conclusions

A reliability-based methodology for serviceability assessment of reinforced concrete structures has been formulated and applied to the deflection assessment of corrosion affected flexural members. With a model of deflection deterioration derived from experimental data, the time-variant probability of serviceability failure can be quantified and the time for the structure to be unserviceable and hence requiring repairs due to excessive deflection can be determined with confidence. Factors that affect the serviceability of corrosion affected concrete structures have also been studied using two sensitivity analysis techniques. It has been found that the deterioration function proposed in the paper can best represent the effect of reinforcement corrosion on deflection deterioration. It has also been found that some variables of concrete structures, such the width of cross-section, B and concrete property, E_c, may be treated as deterministic variables in serviceability assessment if their full statistical information is not available. It can be concluded that time-dependent reliability methods are rational tools for serviceability assessment of corrosion affected concrete structures with a view to determine the time of repairs and/or rehabilitation for the structures.

Acknowledgement

Financial support from the Royal Academy of Engineering, UK with Award No. 10177/93 is gratefully acknowledged.

References

1. ACI 318, (1999), Building Code Requirements for Reinforced Concrete, ACI, Farmington Hills.
2. ACI 365.1R, (2002), "Service Life Prediction – State- of-the-Art Report", ACI Manual of Concrete Practice – Part 5, Farmington Hills.
3. Andrade, C, Alonso, M C. Feliu, S and Gonzalez, J A, (1996), "Advances in the On-Site Electrochemical Measurement of Reinforcement Corrosion and Their Use for Predicting Residual Life", Proc. of 13th Int. Conf. on Corrosion, 3.1 – 3.7, Melbourne.
4. Bentz, B E, Thomas, M.D.A. and Hooton, R. D., (1999), "An Overview and Sensitivity Study of a Multimechanistic Chloride Transport Model", Cement and Concrete Research, 29, 827 – 837.
5. Bjerager, P and Krenk, S, (1987), "Parametric Sensitivity in First Order Reliability Theory", Journal of Engineering Mechanics, ASCE, 115, (7), 1577 – 1582.
6. British Standards BS 8110 (1997) Structural Use of Concrete – Code of Practice for Design and Construction – Part 1, BSI, London.
7. Broomfield, J, (1997), Corrosion of Steel in Concrete, Understanding, Investigating & Repair, E & FN Spon, London.
8. Chaker, V (Ed), (1992), Corrosion Forms & Control for Infrastructure, ASTM STP 1137, Philadelphia.
9. Dhir, R.K. and McCarthy, M.J, (Eds), (1999), Concrete Durability and Repair Technology, Thomas Telford, London.
10. Ditlevsen, O and Madsen, H O, (1996), Structural Reliability Methods, John Wileys & Sons, Chichester, England.
11. Francois, R, and Castel, A, (2001), "Discussion on Influences of Bending Crack and Water-Cement Ratio on Chloride-Induced Corrosion of Main Reinforcing Bars and Stirrups", ACI Materials Journal. 98, (3), 276 – 278.
12. Frangopol, D M, Lin, K Y, and Estes, A, (1997), "Reliability of Reinforced Concrete Girders under Corrosion Attack", J. Struct. Engrg., ASCE, 123, (3), 286 – 297.
13. Gilbert, R I, (1988), Time Effects in Concrete Structures, Elsevier, New York.
14. Li, C.Q. (1995), "Optimisation of Reliability-Based Structural Design", Civil Engrg. Trans., CE37, (4), 303 – 308.
15. Li, C.Q., (2000), "Corrosion Initiation of Reinforcing Steel in Concrete under Natural Salt Spray and Service Loading – Results and Analysis", ACI Materials Journal, 97, (6), 690 – 697.

16. Li, C.Q., (2001), "Initiation of Chloride Induced Reinforcement Corrosion in Concrete Structural Members – Experimentation", ACI Structural Journal, 98, (4), 501 – 510.
17. Li, C.Q., (2003), "Life Cycle Modelling of Corrosion Affected Concrete Structures –Propagation", Journal of Structural Engineering, ASCE, 129, (6), 753 – 761.
18. Li, C.Q., and Melchers R.E., (1993) "Outcrossings from Convex Polyhedrons for Nonstationary Gaussian Processes", J. of Engng Mech., ASCE, 119, (11), 2354 – 2361.
19. Liu, Y., and Weyers, R.E., (1998), "Modeling the Time-to-Corrosion Cracking in Chloride Contaminated Reinforced Concrete Structures", ACI Materials Journal, Nov.-Dec., 675-681.
20. Madsen, H O, Krenk, S, and Lind, N C, (1986), Methods of Structural Safety, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, USA.
21. Melchers, R E, (1999), Structural Reliability Analysis and Prediction, Second Edition, John Wiley and Sons, Chichester.
22. Melchers, R E, (2001), "Assessment of Existing Structures – Approaches and Research Needs", J. Struct. Engrg., ASCE, 127, (4), 406 – 411.
23. Mirza, S A, Hatzinikolas, M, and MacCgregor, J G, (1979), "Statistical Description of Strength of Concrete", J. Struct. Engrg., ASCE, 105, (6), 1021-1037.
24. Novak, D, Teply, B and Shiraishi, N, (1993), "Sensitivity Analysis of Structure: A Review", Proc. 5th Int Conf. Civil & Struct Eng Computing, Edinburgh, 17th – 19th August, 201 – 206.
25. Otsuki, N, Miyazato, S, Diola, N, B and Suzuki, H, (2000), "Influences of Bending Crack and Water-Cement Ratio on Chloride-Induced Corrosion of Main Reinforcing Bars and Stirrups", A.C.I. Materials Journal. 97, (4), 454 – 465.
26. Papoulis, A., (1965) Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York.
27. Schiessl, P., (1988) Corrosion of Steel in Concrete, Report of the Technical Committee 60-CSC RILEM, London, Chapman and Hall.
28. Thoft-Christensen, P and Sorensen, J D, (1987), "Optimal Strategies for Inspection and Repair of Structural System", Civil Engineering System, 4, 94 – 100.